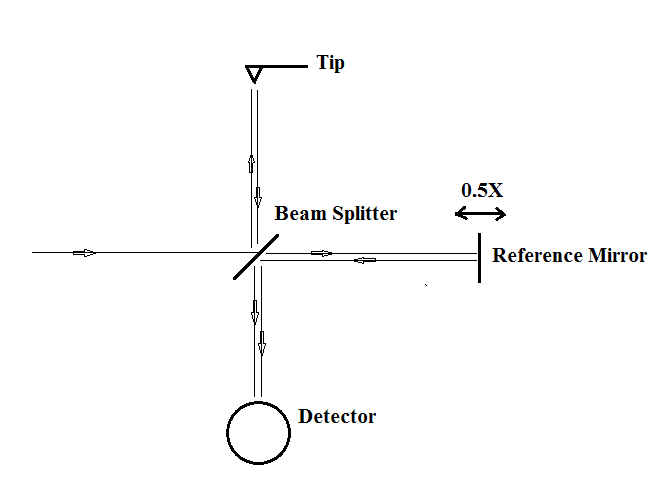
The signal of nano-FTIR



Define the reference beam electric field at the detector as

is the optical path difference introduced by different spatial lengths of the two beams of the interferometer.

is the spatial coordinate of the wave function. Its origin is fixed to the beam splitter of the interferometer. Its value equals the total length of the optical beam goes through after it is split by the beam splitter till it reaches the detector when the two arms of the interferometer have equal length.

is the complex amplitude which satisfies the relation

This is because is a real quantity.

Similarly, the electric field of the signal beam which contains the tip and sample can be defined as

Because is a real quantity, we still have

The complex amplitude can be written as

is the near field signal which is useful. is background signal which is mainly unwanted scattering. The time dependence of the complex amplitude is introduced by the tapping of the AFM tip. This time dependence is a slow one due to the mechanical nature of the oscillation compared with the optical oscillation determined by frequency .

The raw data detected by the detector can be written as

For symmetry reason, it can also be written as

Because and are real quantity, their conjugations are themselves.

2τ is the measuring time of the detector which is related to the speed of the detector.

Define

After carrying out the integral, the following result can be achieved

When we carry out the above integral in time domain due to the average effect of the measuring process, we can fix the in as which is a constant number for the integral and only do the integral for the in the optical oscillation term . The reason is the mechanical oscillation of the tip is much slower than the measuring process of the detector, so within the measuring time 2τ, the complex amplitude does not change. But the optical oscillation determined by is much faster than the measuring process of the detector, so, this term will be averaged out. This simplification is also support by the paper

Alexander A. Govyadinov, Iban Amenabar, Florian Huth, P. Scott Carney, and Rainer Hillenbrand. Quantitative Measurement of Local Infrared Absorption and Dielectric Function with Tip-Enhanced Near-Field Microscopy. *J. Phys. Chem. Lett.* **2013**, *4*, 1526−1531

The notes in reference (26) of above paper explain the problem above very well.

The form of the final result after integration can also be found in the paper

The Determination of Refractive Index Spectra by Fourier Spectrometry. *Infrared Physics*, **1969**, *Vol.* *9*, pp. 185-209

After the raw data at every position of the reference mirror is measured, the signal demodulation is carried out by internal electronics.

We assume the internal electronics does a complex Fourier integral which determined the amplitude and phase of several demodulation orders simultaneously. As a result, we have

Note: All integrations below this formula (containing the above one) go from minus infinity to plus infinity.

If above assumption holds, the will be a complex number, so it can not be plot directly. The interferogram which has been plot in Keilmann and Hillenbrand’s publication (Mid-infrared near-field spectroscopy. *Optics Express.* *Vol. 17,* 21794 (2009) and Infrared spectroscopic nanoimaging with a thermal source. Nature Materials. *Vol.10,* 352 (2011)) may be the amplitude or the real part of the .

Explicitly, the above formula can be written as

The first term in the parentheses corresponds to the reference beam and the second term corresponds to the sample beam, the rest is the cross term of the reference beam and the sample beam.

The first term will vanish after you do the time demodulation, so it will not even appear in the interferogram. The second term will contain the multiplicative background which is the multiplication of electric field of near field signal and background signal. However, since it is x independent, it will vanish after the complex Fourier transform afterwards which is an integral to x. So the first and second term in the parentheses can be removed. Thus there will be no multiplicative background.

Remove the x independent term in , what remains is the interferogram .

Do a complex Fourier transform to this interferogram.

Remember

And

Above formula can be simplified to

Define

If n is large enough, for example, n=2 or 3, we have

This conclusion can be got from the chapter 6.2.6 Near-field vs. Background Scattering, N. Ocelic, Quantitative Near-field Phonon-polariton Spectroscopy, Thesis Technische Universität München.

As a result, there will be no additive background either and the final result can be written as

Where

is the complex effective scattering coefficient which can be calculated from point dipole model or finite monopole model.

is the response function of the parabolic mirror in front of the AFM tip.

is the response function of the beam splitter. It contains the effect of one reflection process and one transmission process. As a result, no matter whether the beam splitter is 50/50, the is the same for both the sample arm and reference arm of the interferometer.

is the complex amplitude of incident wave in front of the beam splitter.

is the response function of the plane mirror at the end of the reference arm.

Where we define

As the total response function of the interferometer

Is the intensity profile of the broadband light source.

Above result manifests that no additive or multiplicative background exists in the broadband nano-FTIR setup. There is no multiplicative background because the term contains it is x independent and will vanish after the complex Fourier transform to x. There is no additive background because the higher harmonics of background electric field is much smaller than the near field electric field, thus the former can be ignored.

Supplementary

According to the page 4, we have the formula for the interferogram *In* (*x*) with the reference beam mirror at the position *x*:

Define

,

,

corresponds to the measurement time which lasts a period of time of *T*.

is the initial time the detection begins at while the mirror stays at the position . *Δt* is the piezo movement time every step while *Δx* is the step length of the piezo movement.

According to

page 7

page 6

is the effective scattering factor of the whole tip including apex and shaft while other quantities’ definitions keep the same.

The realistic signals detected with source fluctuation should be

)

is the amplitude temporal modulation at much slower scale than optical oscillation.

Define

is randomized by thermal collisions.

However, solely

remains in the final result.

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